Chapter 4 Lecture - Introduction to Valuation: The Time Value of Money

Chapter 4 Lecture - Introduction to Valuation: The Time Value of Money

Learning Objectives

After studying this chapter, you should be able to:

LO1 Determine the future value of an investment made today.

LO2 Determine the present value of cash to be received at a future date.

LO3 Calculate the return on an investment.

LO4 Predict how long it takes for an investment to reach a desired value.

4-2

Basic Definitions

- **Present Value (PV)**
  - **The current value of future cash flows discounted at the appropriate discount rate**
  - **Value at t=0 on a time line**
- **Future Value (FV)**
  - **The amount an investment is worth after one or more periods.**
  - **"Later" money on a time line**

Basic Definitions

Interest rate (r or *i*)

- **Discount rate - the rate of return used to discount future cash flows back to their present value.**
- **Cost of capital - the required return necessary to make a capital budgeting project, such as building a new factory, worthwhile.**
- **Opportunity cost of capital - the incremental return on investment that a business foregoes when it elects to use funds for an internal project, rather than investing cash in a marketable security. ...**
- **Required return - the required return is the minimum amount of profit (return) an investor will receive for assuming the risk of investing in a stock or another type of security.**

- **Terminology depends on usage**

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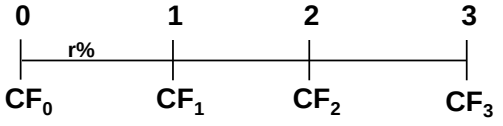
Time Line of Cash Flows

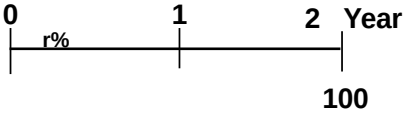
- Tick marks at ends of periods
  - Time 0 is today;
  - Time 1 is the end of Period 1



+CF = Cash INFLOW -CF = Cash OUTFLOW PMT (payment) = Constant CF

4-5

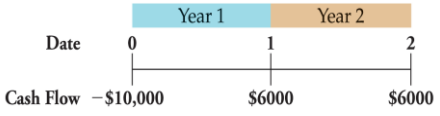
Time Line for a $100 Lump Sum due at the End of Year 2.



4-6

The Timeline Example (cont'd)

- Assume that you are lending $10,000 today and that the loan will be repaid in two annual $6,000 payments. This assumes some interest.



The first cash flow at date 0 (today) is represented as a negative sum because it is an outflow.

- Timelines can represent cash flows that take place at the end of any time period.

4-7

Simple Interest and Compound Interest

- What is the difference between simple interest and compound interest?
  - Simple interest: Interest is earned only on the principal amount.
  - Compound interest: Interest is earned on both the principal and accumulated interest of prior periods.

- Example: Suppose that you deposit $500 in your savings account that earns 5% annual interest. How much will you have in your account after two years using (a) simple interest and (b) compound interest?

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Example

- Simple Interest
  - Interest earned = 5% of $500 = .05×500 = $25 per year
  - Total interest earned = $25×2 = $50
  - Balance in your savings account:
  - = Principal + accumulated interest
    = $500 + $50 = $550
- Compound interest (assuming compounding once a year)
  - Interest earned in Year 1 = 5% of $500 = $25
  - Interest earned in Year 2 = 5% of ($500 + accumulated interest)
    = 5% of ($500 + 25) = .05×525 = $26.25
  - Balance in your savings account:
    = Principal + interest earned = $500 + $25 + $26.25
    = $551.25

Future Values: General Formula

$$FV = PV(1 + r)^t$$

FV = future value
PV = present value
r = period interest rate, expressed as a decimal
t = number of periods

- Future value interest factor = $(1 + r)^t$

Future Values – Example

- Suppose you invest $100 for one year at 10% per year. What is the future value in one year?
  - Interest = 100(.10) = 10
  - Value in one year
    = Principal + interest
    = 100 + 10 = 110
  - Future Value (FV)
    = 100(1 + .10) = 110
- Suppose you leave the money in for another year. How much will you have two years from now?

FV = 100(1.10)(1.10)
= 100(1.10)^2 = 121.00

Future Values – Another Example

- Suppose you invest the $100 from the previous example for 5 years. How much would you have?

Formula Solution:

FV =PV(1+r)^t
=100(1.10)^5
=100(1.6105)
=161.05

TABLE 4.1
Future value of $100 at 10 percent

Year	Beginning Amount	Interest Earned	Ending Amount
1	$100.00	$10.00	$110.00
2	110.00	11.00	121.00
3	121.00	12.10	133.10
4	133.10	13.31	146.41
5	146.41	14.64	161.05
		Total interest $61.05	

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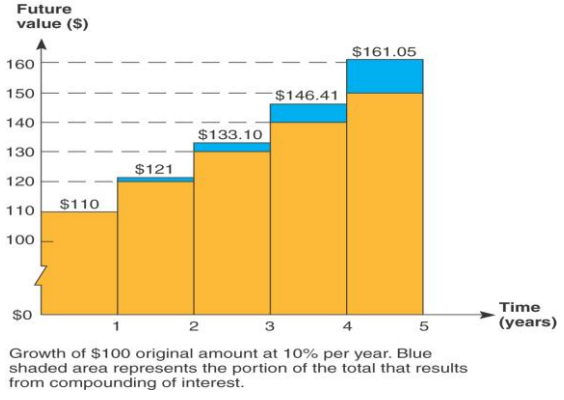
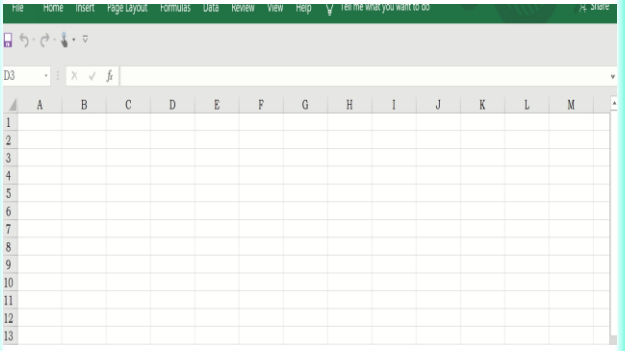


Growth of $100 original amount at 10% per year. Blue shaded area represents the portion of the total that results from compounding of interest.

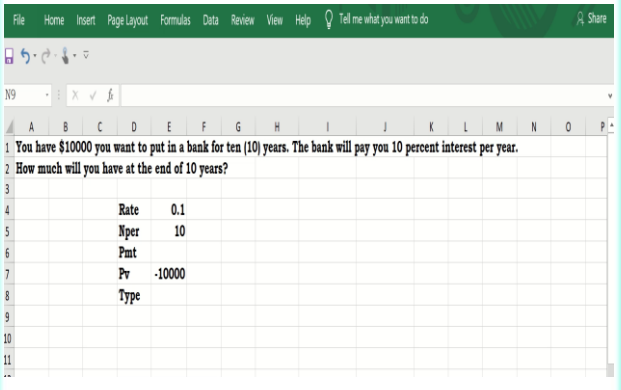
4-13

Brief Introduction to Excel – Scroll down to see PLAY



4-14

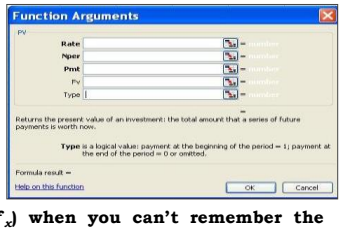
Calculating FV in Excel



4-15

Excel Spreadsheet Functions

- **Excel TVM functions:**
  =FV(rate,nper,pmt,pv)
  =PV(rate,nper,pmt,fv)
  =RATE(nper,pmt,pv,fv)
  =NPER(rate,pmt,pv,fv)



- **Use the formula icon (f_x) when you can't remember the exact formula**

Excel Example

Rate	0.1		
Nper	5		
Pmt	0		
PV	-100	FV	161.05

4-16

Some Calculators

Time Value of Money Calculators
https://www.calculatorsoup.com/calculators/financial/index-time-value-of-money-calculators.php

Online Financial Calculator
http://www.grunderware.com

Instructions for using Texas Instruments BA II Plus Calculator
Manual Quick Reference Tutorial

BAII Plus™ Financial Calculator App for iOS® devices
http://education.ti.com/en/us/products/apps/baii-plus-iphone-app/features/features-summary

4-17

Future Values – One More Example

Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?

Formula Solution:

$FV = PV(1+r)^t$

$= 10(1.055)^{200}$

$=10(44718.984)$

$=447,189.84$

Excel Solution:

=FV(Rate,Nper,Pmt,PV)

=FV(0.055,200,0,10)

= -447189.84

NOTE: Rate = decimal

Calculator Solution

200	N
5.5	I/Y
10	PV
0	PMT
CPT FV	=
-447,189.84	

Excel Solution

Rate	0.055		
Nper	200		
Pmt	0		
PV	10	FV	(447,189.84)

4-18

Future Value: General Growth Formula

Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years?

Formula Solution:

$FV = PV(1+r)^t$

$=3(1.15)^5$

$=3(2.0114)$

$=6.0341$ million

Excel Solution:

=FV(Rate,Nper,Pmt,PV)

=FV(0.15,5,0,-3000000)

= 6.0341

Calculator Solution

5	N
15	I/Y
-3	PV
0	PMT
CPT FV	=
6.0341	

Excel Solution

Rate	0.15		
Nper	5		
Pmt	0		
PV	-3000000	FV	6,034,071.56

4-19

Future Value: Important Relationship I

For a given interest rate:

- **The longer the time period,**
- **The higher the future value**

For a given r, as t increases, FV increases

$$FV = PV(1 + r)^t$$

Future Value: Important Relationship II

For a given time period:

- **The higher the interest rate,**
- **The larger the future value**

For a given t, as r increases, FV increases

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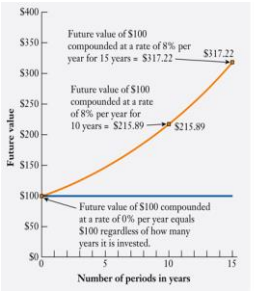
Power of Time

Future Value and Compound Interest Illustrated



The Power of Time

This figure illustrates the importance of time when it comes to compounding. Because interest is earned on past interest, the future value of $100 deposited in an account that earns 8% compounded annually grows over threefold in 15 years. If we were to expand this figure to 45 years (which is about how long you have until you retire, assuming you're around 20 years old right now), it would grow to over 31 times its initial value.

Future value of original investment increases with time, unless interest rate is zero.

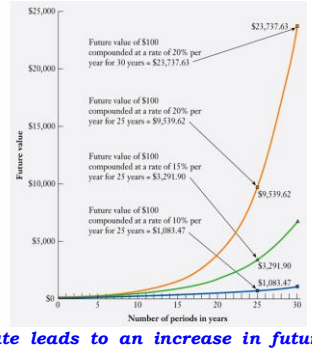
4-21

Power of Interest Rate

Future Value and Compound Interest Illustrated

The Power of the Rate of Interest

This figure illustrates the importance of the interest rate in the power of compounding. As the interest rate climbs, so does the future value. In fact, when we change the interest rate from 10% to 20%, the future value in 25 years increases by over 8 times, jumping from $1,083.47 to $9,539.62.



An increase in interest rate leads to an increase in future value.

4-22

Quick Quiz

- **What is the difference between simple interest and compound interest?**
- **Suppose you have $500 to invest and you believe that you can earn 8% per year over the next 15 years.**
  - **How much would you have at the end of 15 years using compound interest?**

=FV(.08, 15, 0, -500)

Excel Solution

Rate	0.08		
Nper	15		
Pmt	0		
PV	-500	FV	1,586.08

  - **How much would you have using simple interest?**

500 + 15(500)(.08) = 1,100

4-23

Present Values

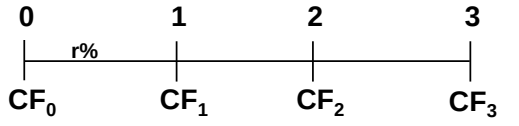
- **The current value of future cash flows discounted at the appropriate discount rate**
- **Value at t=0 on a time line**
- **Answers the questions:**
  - **How much do I have to invest today to have some amount in the future?**
  - **What is the current value of an amount to be received in the future?**
- ***Present Value = the current value of an amount to be received in the future***
- **Why is it worth less than face value?**
  - **Opportunity cost**
  - **Risk & Uncertainty**

Discount Rate = *f* (time, risk)

4-24

Time Line of Cash Flows

- Tick marks at ends of periods
  - Time 0 is today;
  - Time 1 is the end of Period 1



Remember	+CF = Cash INFLOW	-CF = Cash OUTFLOW

4-25

Present Values

- We know $FV = PV(1 + r)^t$
- Rearrange to solve for PV

$$PV = FV / (1+r)^t$$

$$\text{or } PV = FV(1+r)^{-t}$$

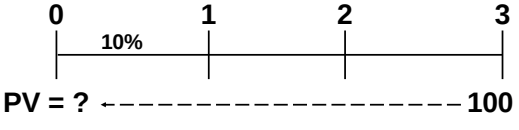
- "Discounting" = finding the present value of one or more future amounts.

4-26

What's the PV of $100 due in 3 Years if r = 10%?

Finding PVs is discounting, and it's the reverse of compounding.

Excel Solution			
Rate	0.1		
Nper	3		
Pmt	0		
Fv	100	PV	(75.13)

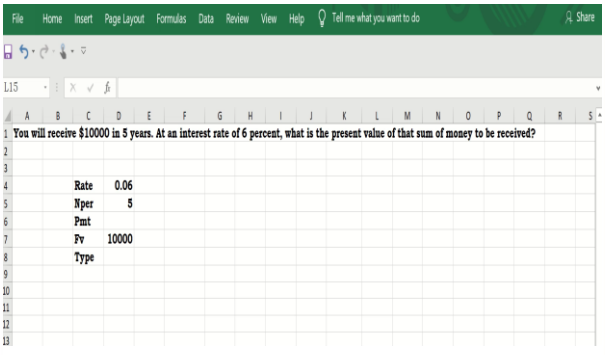


Formula: $PV = FV(1+r)^{-t} = 100(1.10)^{-3} = \$75.13$

Excel: =PV(.10,3,0,100) = -75.13

4-27

Calculating PV in Excel



4-28

Present Value: Example of Single Period

Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

Formula Solution:

$PV = FV(1+r)^{-t}$

$=10{,}000(1.07)^{-1}$

$=10{,}000/1.07$

$=9{,}345.79$

Excel Solution:

=PV(Rate,Nper,Pmt,FV)

=PV(0.07,1,0,10000)

`= -9345.79

Calculator Solution

1 N
7 I/Y
0 PMT
10000 FV
CPT PV =
-9345.79

Excel Solution

Rate	0.07		
Nper	1		
Pmt	0		
Fv	10000	PV	(9,345.79)

4-29

Present Values: Example Multi-Periods

You want to begin saving for your daughter's college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?

Formula Solution:

$PV = FV(1+r)^{-t}$

$=150{,}000(1.08)^{-17}$

$=150{,}000/(1.08)^{17}$

$=40{,}540.34$

Excel Solution:

=PV(Rate,Nper,Pmt,FV)

=PV(0.08,17,0,150000)

= -40,540.34

Calculator Solution:

17 N
8 I/Y
0 PMT
150000 FV
CPT PV =
-40,540.34

Excel Solution

Rate	0.08		
Nper	17		
Pmt	0		
Fv	150000	PV	(40,540.34)

4-30

Present Values: Another Example of Multi-Periods

Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?

Formula Solution:

$PV = FV(1+r)^{-t}$

$=19{,}671.51(1.07)^{-10}$

$=19.671.51/(1.07)^{10}$

$=-10{,}000$

Excel Solution:

=PV(Rate,Nper,Pmt,FV)

=PV(0.07,10,0,19671.51)

= -10000

Calculator Solution:

10 N
7 I/Y
0 PMT
19671.51 FV
CPT PV
= -10000

Excel Solution

Rate	0.07		
Nper	10		
Pmt	0		
Fv	19671.5	PV	(10,000.00)

4-31

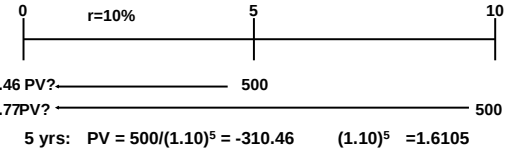
Present Value: Important Relationship I

For a given interest rate:

- **The longer the time period,**
- **The lower the present value**

$$PV = \frac{FV}{(1+r)^t}$$

What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%



5 yrs: $PV = 500/(1.10)^5 = -310.46$ $(1.10)^5 = 1.6105$

10 yrs: $PV = 500/(1.10)^{10} = -192.77$ $(1.10)^{10} = 2.5937$

For a given r, as t increases, PV decreases

4-32

Present Value: Important Relationship II

For a given time period:
- **The higher the interest rate,**
- **The smaller the present value**

$$PV = \frac{FV}{(1+r)^t}$$

What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?

We can show that if the interest rate is 10%, PV = 310.46
We can show that if the interest rate is 15%, PV = 248.59

For a given t, as r increases, PV decreases

4-33

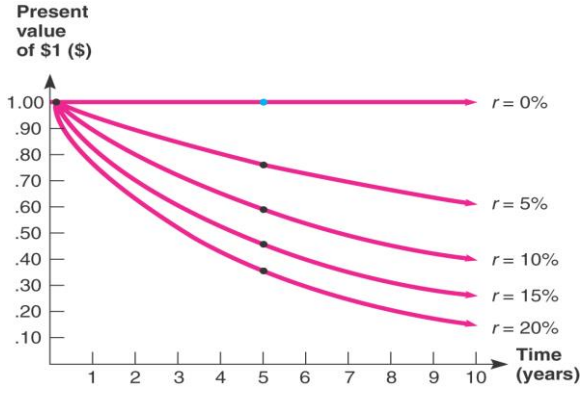
Present Value: Important Relationship II

What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?

Rate = 10%	
Calculator Solution:	
5	N
10	I/Y
0	PMT
500	FV
CPT PV = -310.46	

Rate = 15%	
Calculator Solution:	
5	N
15	I/Y
0	PMT
500	FV
CPT PV = -248.59	

4-34



4-35

Quick Quiz

- **What is the relationship between present value and future value?**
- **Suppose you need $15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?**

Excel Solution

Rate	0.06		
Nper	3		
Pmt	0		
Fv	15000	PV	(12,594.29)

- **If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?**

Excel Solution

Rate	0.08		
Nper	3		
Pmt	0		
Fv	15000	PV	(11,907.48)

Difference = $686.81

4-36

The Basic PV Equation - Refresher

$$PV = FV / (1 + r)^t$$

There are four parts to this equation

- PV, FV, r and t
- Know any three, solve for the fourth

- **Be sure and remember the sign convention**

+CF = Cash INFLOW -CF = Cash OUTFLOW

4-37

Discount Rate

- **To find the implied interest rate, rearrange the basic PV equation and solve for r:**

$$FV = PV(1 + r)^t$$

$$r = (FV / PV)^{1/t} - 1$$

You are looking at an investment that will pay $1200 in 5 years if you invest $1000 today. What is the implied rate of interest?

Formula: r = (1200 / 1000)1/5 – 1 = .03714 = 3.714%

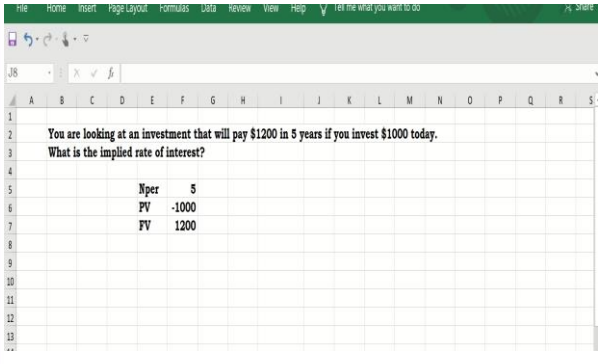
Excel: =RATE(5,0,-1000,1200) = 0.03714

Excel Solution

Nper	5	
Pmt	0	
Pv	-1000	
Fv	1200	Rate 0.03714

4-38

Calculating Rate in Excel



4-39

Discount Rate With Calculator

You are looking at an investment that will pay $1200 in 5 years if you invest $1000 today. What is the implied rate of interest?

Calculator – the sign convention matters!!!

5	N
-1000	PV (you pay $1,000 today)
0	PMT
1200	FV (you receive $1,200 in 5 years)

CPT I/Y = 3.714%

4-40

Discount Rate – Some More Examples

Suppose you are offered an investment that will allow you to double your money in 6 years. You have $10,000 to invest. What is the implied rate of interest?

Excel: =RATE(6,0,-10000,20000) = 0.1225

Calculator: 6 N, -10000 PV, 0 PMT, 20000 FV, CPT I/Y = 12.25%

Suppose you have a 1-year old son and you want to provide $75,000 in 17 years towards his college education. You currently have $5,000 to invest. What interest rate must you earn to have the $75,000 when you need it?

Excel: =RATE(17,0,-5000,75000) = 0.1727

Calculator: 17 N, -5000 PV, 0 PMT, 75000 FV, CPT I/Y = 17.27%

4-41

Quick Quiz

- What are some situations in which you might want to compute the implied interest rate?
- Suppose you are offered the following investment choices:
  - You can invest $500 today and receive $600 in 5 years. The investment is considered low risk.
  - You can invest the $500 in a bank account paying 4% annually.
  - What is the implied interest rate for the first choice and which investment should you choose?

Excel: =RATE(Nper, Pmt, Pv, Fv) = RATE(5, 0, -500, 600) = 3.71%

Excel Solution

Nper	5		
Pmt	0		
Pv	-500		
Fv	600	Rate	0.03714

Calculator: 5 N, -500 PV, 0 PMT, 600 FV, CPT I/Y = 3.714

Put money in the bank

4-42

Finding the Number of Periods

Start with basic equation and solve for t:

$FV = PV(1 + r)^t$

$$\ln FV = \ln PV + t \ln(1+r)$$
$$t \ln(1+r) = \ln FV - \ln PV$$
$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

Excel: = NPER(Rate, Pmt, PV, FV)

Calculator: CPT N

4-43

Number of Periods – Example

You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?

Formula Solution:

FV/PV = 20,000/15,000 = 1.333
ln(1.333) = 0.2877
ln(1.10) = 0.0953
t = 0.2877/0.0953 = 3.0189

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

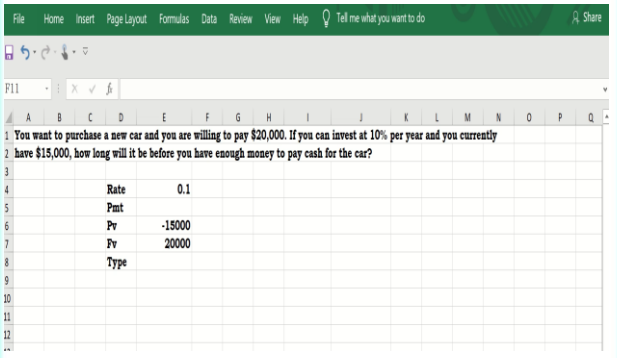
Excel: =NPER(0.10,0,-15000,20000) = 3.018

Excel Solution

Rate	0.1		
Pmt	0		
Pv	-15000		
Fv	20000	NPER	3.01838

4-44

Calculating Nper in Excel



4-45

Rule of 72

- **Rule of 72 is an approximate formula to determine the number of years it will take to double the value of your investment.**
- **Rule of 72:**

N = 72/interest rate in percentage

- **Example: Using Rule of 72, determine how long it will take to double your investment of $10,000 if you are able to generate an annual return of 9%.**
  - **Exact N=ln(2)/ln(1.09)=0.693/0.086=8.04**
  - **Approximate N=72/9=8.**

http://www.moneychimp.com/features/rule72.htm

4-46

Quick Quiz

- **When might you want to compute the number of periods?**
- **Suppose you want to buy some new furniture for your family room. You currently have $500 and the furniture you want costs $600. If you can earn 6%, how long will you have to wait if you don't add any additional money?**

t = ln(600/500) / ln(1.06) = 3.13 years

Excel Solution

RATE(Nper, 0, -500, 600)

Rate	0.06		
Pmt	0		
Pv	-500		
Fv	600	NPER	3.12897

Calculator: 6 I/Y -500 PV 0 PMT 600 FV CPT N
= 3.13 years

4-47

I. Symbols
PV = Present value, what future cash flows are worth today
FV_t = Future value, what cash flows are worth in the future
r = Interest rate, rate of return, or discount rate per period typically , but not always, one year
t = Number of periods typically , but not always, the number of years
C = Cash amount

II. Future value of C invested at r percent per period for t periods
$FV_t = C \times (1 + r)^t$
The term $(1 + r)^t$ is called the *future value factor.*

III. Present value of C to be received in t periods at r percent per period
$PV = C/(1 + r)^t$
The term $1/(1 + r)^t$ is called the *present value factor.*

IV. The basic present value equation giving the relationship between present and future value is
$PV = FV_t/(1 + r)^t$

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