Perpetuities

A **perpetuity** is a series of equal payments over an infinite time period into the future. Consider the case of a cash payment *C* made at the end of each year at interest rate *i*, as shown in the following time line:

Perpetuity Time Line

0 1 2 3

. . .

PV C C C

Because this cash flow continues forever, the present value is given by an infinite series:

$$PV = C / (1 + i) + C / (1 + i)^2 + C / (1 + i)^3 + \ldots$$

From this infinite series, a usable present value formula can be derived by first dividing each side by (1 + *i*).

$$PV / (1 + r) = C / (1 + r)^2 + C / (1 + r)^3 + C / (1 + r)^4 + \ldots$$

In order to eliminate most of the terms in the series, subtract the second equation from the first equation:

$$PV - PV / (1 + r) = C / (1 + r)$$

Solving for *PV*, the present value of a perpetuity is given by:

$$\boldsymbol{PV = \frac{C}{r}}$$

Growing Perpetuities

Sometimes the payments in a perpetuity are not constant but rather, increase at a certain growth rate *g* as depicted in the following time line:

Growing Perpetuity Time Line

0 1 2 3

. . .

PV C C(1+g) $C(1+g)^2$

The present value of a growing perpetuity can be written as the following infinite series:

$$PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots$$

To simplify this expression, first multiply each side by $(1 + g) / (1 + r)$:

$$\frac{PV(1+g)}{(1+r)} = \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots$$

Then subtract the second equation from the first:

$$PV - \frac{PV(1+g)}{(1+r)} = \frac{C}{(1+r)}$$

Finally, solving for *PV* yields the expression for the present value of a growing perpetuity:

$$\boldsymbol{PV = \frac{C}{r - g}}$$

For this expression to be valid, the growth rate must be less than the interest rate, that is, $g < r$.