REVIEW OF UTILITY THEORY AND INDIFFERENCE CURVES

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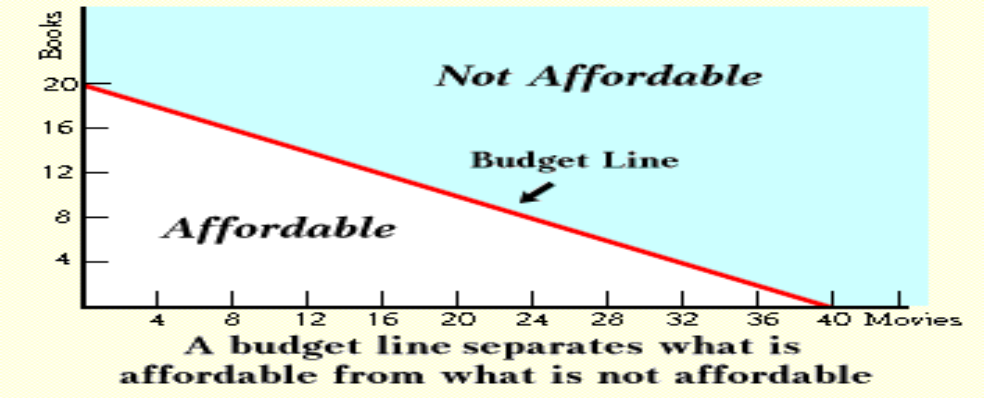
Our objective is to construct a simple model of consumer behavior that will permit us to predict consumers' reactions to changes in their opportunities and constraints. We will take tastes and preferences as given, but we will represent them with a very general analytical model.

We will First Look at the Consumer's Opportunity Set or Budget Constraint

General Formulation: Assume for the moment there are only two goods in the world -- this is a simplification we will relax later -- a person therefore will spend all his income on these two goods.

This can be depicted as $Y = P_X Q_X + P_Y Q_Y$ where the first term is the person's total expenditure on X and the second term the total expenditure on Y.

Let's graph this using some numbers, suppose I = \$200, P_X = \$5, and P_Y = \$10 -- note, as is usually the case, this person is a price taker. From this information we can graph a budget constraint.



What is the maximum amount of X (Movies) this person can buy? 40 units. How does one know that? From the general formula, (Income or Y)/P_X .

a) What is the maximum amount of Y this person can buy? 20 units.
b) Suppose she wants one unit of X -- now what is the maximum amount of Y she could purchase? It would be 18 -- so when you buy ONE X, you give up TWO Y -- this is the REAL or RELATIVE price of X.

Of course we can put this into a general formula, as follows:

$I = P_X X + P_Y Y$ $\quad P_Y Y = I - P_X X$ or $Y = \frac{I}{P_Y} - \left(\frac{P_X}{P_Y}\right) X$ The first term is the intercept of the Y-axis; the second term is the slope. Note **the slope is the price of X over the price of Y -- the relative price or $-\frac{P_X}{P_Y}$.**

We noted that anything within (below and to the left) of the budget line is obtainable; anything beyond the budget line is not obtainable. In this world where we are spending all our income, we will always be on the budget line.

Changes in the Budget Constraint.

There are three givens when we construct the budget line, income, price of X, and price of Y -- if any of these things change, the budget line changes. What happens if:

a) Income increases?
b) Price of X goes up?
c) Price of Y goes down?
d) Given a gift certificate of $20 for X?

Utility: theoretical concept that represents the level of satisfaction or enjoyment that a consumer receives from consumption of a good.

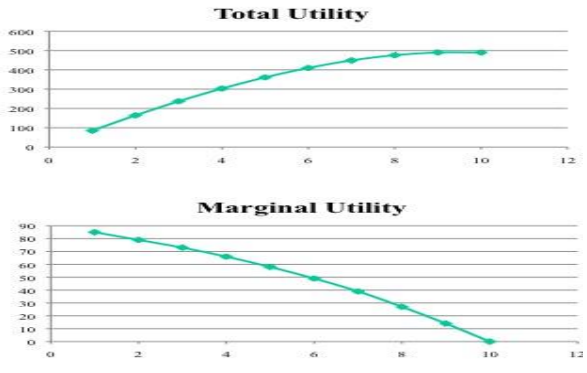
We do not measure utility. Consumers do not measure their utility in any units of measure, but they can rank their utilities from different consumption bundles.

For the moment, define Total Utility as the utility that a consumer receives from all of the units of a particular good that she consumes. Define Marginal Utility as the increase in Total Utility that corresponds to a one-unit increase in consumption of a good.

Diminishing Marginal Utility: plays a very important role in our analysis of consumer behavior.

As the quantity of a good consumed increases (ceteris paribus), the marginal utility attached to consuming additional units of the good eventually diminishes.

Quantity	Marginal Utility	Total Utility
1	85	85
2	79	164
3	73	237
4	66	303
5	58	361
6	49	410
7	39	449
8	27	476
9	14	490
10	0	490



The First Steps in Understanding Utility – Make some assumptions about people/individuals.

Assumption One: **Completeness**.

We assume that an individual has *preferences over any two bundles of goods* (a bundle can be a single good or a bunch of goods). In other words they can choose between them or decide they are indifferent. Prefer A to B or Prefer B to A or be Indifferent

Assumption Two: **More is better**.

This is simply that if a person considers some thing to be a good (i.e. they value it) then more of it is preferred to less.

From these two assumptions alone we can construct an indifference curve.

An **Indifference Curve** is a line (curve) that shows all the possible combinations of two goods between which a person is indifferent. In other words, it shows the consumption of different combinations of two goods that will give the same utility (satisfaction) to the person.

INDIFFERENCE CURVE: Suppose the consumer can achieve a given level of utility with various combinations of goods X and Y: a lot of X and little Y; a medium amount of X and a medium amount of Y; a little X and a lot of Y. These combinations all give the same level of utility. One way to illustrate this idea is the indifference curve in the following diagram:

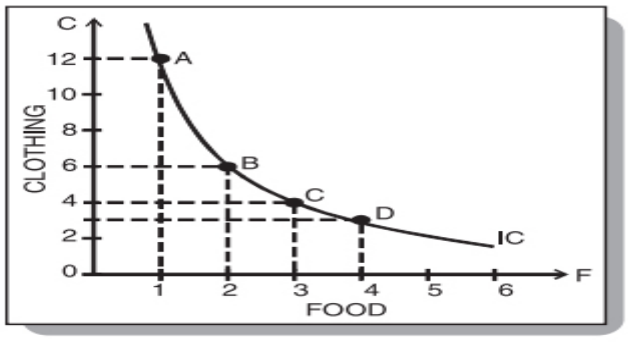


Fig. 1 : A Consumer's Indifference Curve

Higher levels of utility can be illustrated by indifference curves that lie to the NorthEast of U_0. We know they are higher levels of utility because "more is better."

Indifference Map

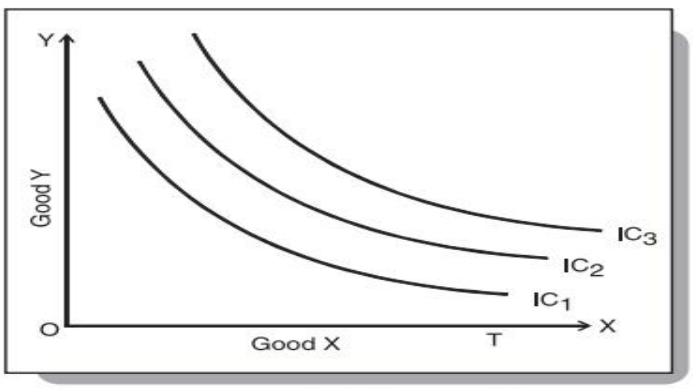


Fig. 2 : Indifference Map

Indifference curves have three properties:

1. Slope down because "more is better" and to maintain constant utility while reducing consumption of one good, the consumption of the other good must be increased.

2. Convex to the origin because of the Law of Diminishing Marginal Utility.

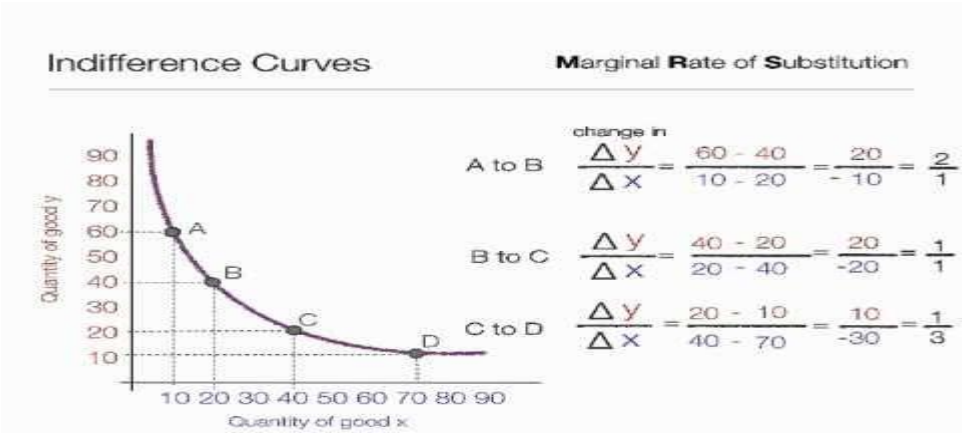
Along an indifference curve, we say that **utility is constant** -- in other words, if I am indifferent between two bundles that means it gives me the same level of utility.

Therefore we have the following: $MU_X\Delta X + MU_Y\Delta Y = 0$

The first terms is the addition in utility resulting from additional X (so $\Delta X>0$); the second term is the decrease in utility resulting from the decrease in Y (so $\Delta Y<0$).

Rearranging $MU_X\Delta X = -MU_Y\Delta Y$.

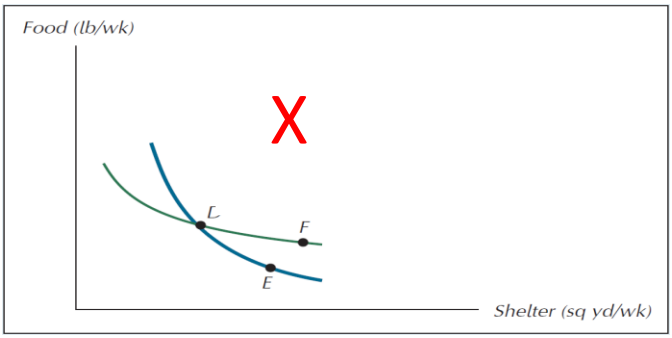
The slope of the indifference curve is called the marginal rate of substitution, which is $\Delta Y/\Delta X$ (rise over the run). So as you can see $MRS = \frac{\Delta Y}{\Delta X} = -\frac{MU_X}{MU_Y}$. **So the marginal rate of substitution is simply the ratio of the marginal utilities.**



Note: as we slide down the indifference curve, we enjoy more X and less Y: MU_X must be decreasing and MU_Y must be increasing: thus the slope of the indifference curve is decreasing. The indifference curve is convex to the origin.

As we said before, the ratio of marginal utilities has a name: the Marginal Rate of Substitution (MRS). Sometimes we say "MRS of X for Y" but this can be ambiguous. Say "MRS = this amount of Y relinquished to gain an additional unit of X."

3. Indifference curves cannot intersect (preference orderings are transitive). We can see why indifference curves cannot intersect by considering a case in which they do intersect and observing a contradiction.



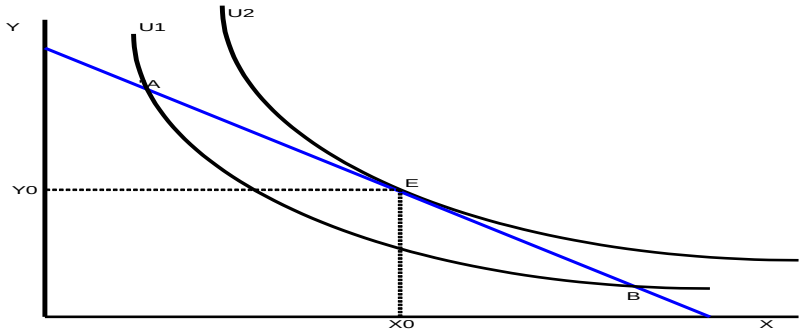
Consumer Equilibrium

We have defined indifference curves as curves of constant utility, whose slopes reflect the rate at which consumers are willing to trade one good for another. This rate is formalized in the Marginal Rate of Substitution, which is equal to (minus) the slope of the indifference curve and equal to the ratio of marginal utility of X to marginal utility of Y.

We have defined a consumer's budget constraint as the straight line that describes all combinations of the two goods that the consumer can afford. The budget constraint has slope equal to (minus) the ratio of the price of X to the price of Y.

The question of consumer equilibrium is: which of the points on the budget constraint (that the consumer can afford) will the consumer select?

The consumer will maximize income by moving along the budget constraint until she reaches the highest possible indifference curve. This will be the one that is just tangent to the budget constraint.

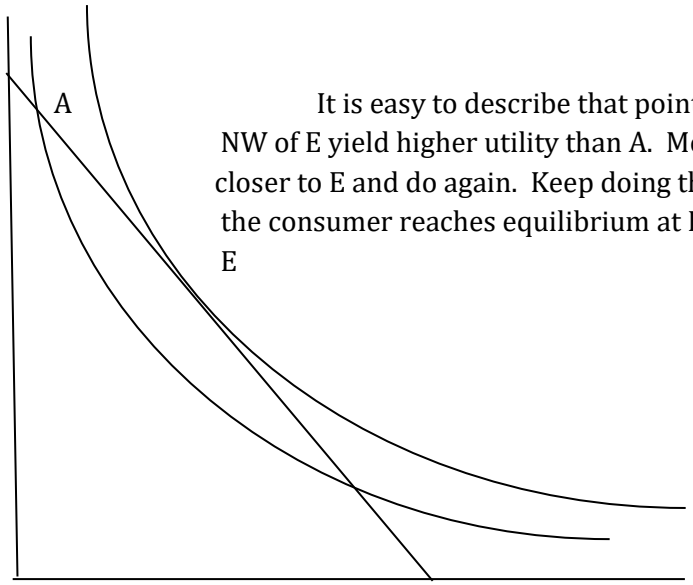


At the tangency point, the slope of the indifference curve and the slope of the budget constraint are identical. This gives us the consumer's equilibrium condition:

$$MRS_{XY} = MU_X/MU_Y = P_X/P_Y \qquad \text{Rearrange to see} \qquad MU_X/P_X = MU_Y/P_Y.$$

In other words, as we saw before, the consumer can maximize utility by equalizing marginal utility per unit of currency spent across goods.

Consider a situation in which the consumer's equilibrium condition is not met:



It is easy to describe that points SE of A and NW of E yield higher utility than A. Move A closer to E and do again. Keep doing this until the consumer reaches equilibrium at E.

Example: Let Mary's weekly expenditure be \$10.00 and P_N = \$1.00 and P_M= \$2.00

N = newspapers, M= magizines

N	U(N)	MU(N)	MU(N)/PN	M	U(M)	MU(M)	MU(M)/PM
0	0			0	0		
		12	12			20	10
1	12			1	20		
		8	8			12	6
2	20			2	32		
		6	6			8	4
3	26			3	40		
		4	4			4	2
4	30			4	44		
		2	2			2	1
5	32			5	46		

Prove Mary will buy 4 Newspaper and 3 Magazines