Bayes' Rule Examples

1. Let us say P(Fire) means how often there is fire, and P(Smoke) means how often we see smoke, then:

P(Fire|Smoke) means how often there is fire when we can see smoke)
P(Smoke|Fire) means how often we can see smoke when there is fire

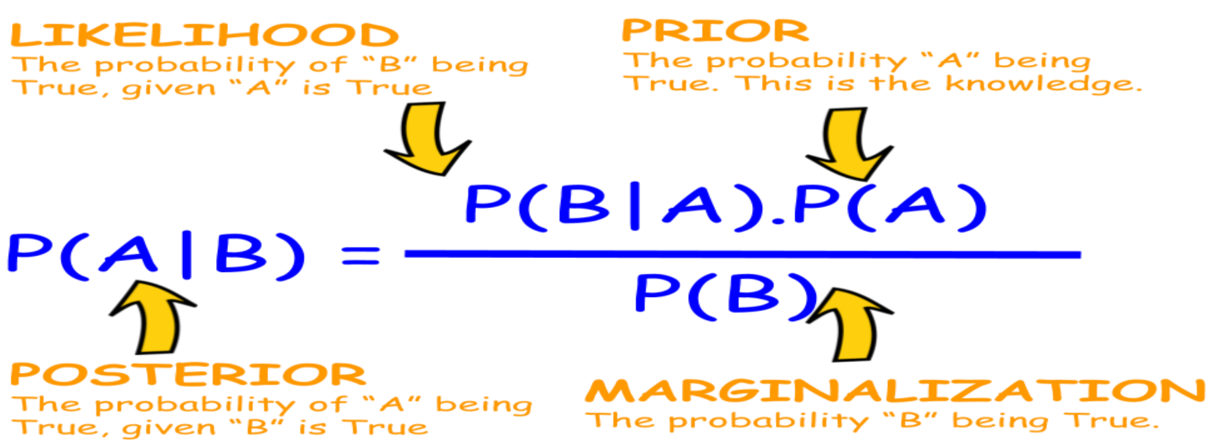
So the formula kind of tells us "forwards" P(Fire|Smoke) when we know "backwards" P(Smoke|Fire)

We know:

- dangerous fires are rare so P(Fire) =1%
- but smoke is fairly common so P(Smoke) = 10% due to barbecues, for example.
- and 90% of dangerous fires make smoke so P(Smoke/Fire) = 90%

We can then discover the **probability of dangerous Fire when there is Smoke**:

Using Bayes' Formula



P(Fire|Smoke) =*P(Fire) P(Smoke|Fire)***P(Smoke)** =*1% x 90%***10%** =9%

So it is still worth checking out any smoke to be sure.

2. You are interested in finding out a patient's probability of having liver disease if they are an alcoholic. "Being an alcoholic" is the **test** (kind of like a litmus test) for liver disease.

   - **A or (LD)** could mean the event "Patient has liver disease." Past data tells you that 10% of patients entering your clinic have liver disease. P(LD) = 0.10.
   - **B or (AL)** could mean the litmus test that "Patient is an alcoholic." Five percent of the clinic's patients are alcoholics. P(AL) = 0.05.
   - You might also know that among those patients diagnosed with liver disease, 7% are alcoholics. This is your **B/A or AL/LD:** the probability that a patient is alcoholic, given that they have liver disease, is 7%.

You want to find out the probability of having liver disease, given that one is an alcoholic.

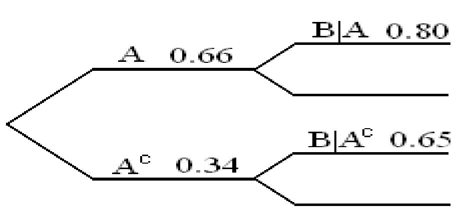
Bayes' theorem tells you:

P(A|B) = [P(B|A) * P(A)] / P(B)   or P(LD|AL) = [P(AL|LD) * P(AL)] / P(LD)

P(AL|LD) = (0.07 * 0.1)/0.05 = 0.14

3. The effect of an antidepressant drug varies from person to person. Suppose that the drug is effective on 80% of women and 65% of men. It is known that 66% of the people who take the drug are women.
   a) What is the probability that the drug is effective for a person taking the drug? Use a probability tree to answer this question.
   b) Suppose that you are told that the drug is effective. What is the probability that the drug taker is a man?
   Hint: rely on values from the probability tree to answer this question.

      a) Define events: A = woman, B = drug is effective



Joint events	Probabilities
A and B	(0.66)(0.80) = 0.528
A^C and B	(0.34)(0.65) = 0.221

P(EFFECTIVE) =.0528+.221=.749

      b) $P(A^C \mid B) = \frac{P(A^C \text{ and } B)}{P(B)} = \frac{.221}{.749} = .295$

4. Imagine you come to the hospital to test whether you have COVID-19 or not. You receive the bad news: you have tested positive for COVID! However, you think that there could be a chance that the result is wrong.

By doing some research on the internet, you know three pieces of information:

- the probability that a person has a positive test result given that person is healthy is 0.01
- the probability that a patient has a positive test result given that person has COVID is 0.98
- the probability that a patient has COVID is 0.01

Note: these numbers are not real and are made up for teaching purposes.

What is the probability that a patient has COVID if that person tested positive for COVID?

Hint: you will need to apply Bayes' rule. Show all steps of your analysis and calculation.

Step 1: Define the events

Let A be the event of actually having COVID-19:

- A1: the patient has COVID-19
- A2: the patient does not have COVID-19

Let B be the testing result for COVID-19

- B1: the patient tested positive for COVID-19
- B2: the patient did not test positive for COVID-19

Step 2: Define given probabilities

From the question, we know that:

- P(B1|A2) = 0.01 P(B1|A1) = 0.98 P(A1) = 0.01 P(A2) = .99

Step 3: Define the probability that we need to find

We need to find the probability that a patient has COVID-19 if he tested positive for COVID-19. In other words, we need to find:

P(A1|B1) = ?

Step 4: apply the formula and find the probability in question

$$P(A1|B1) = \frac{P(A1)P(B1|A1)}{P(A1)P(B1|A1)+P(A2)P(B1|A2)} = \frac{0.01*0.98}{0.01*0.98+0.99*0.01} = \frac{0.0098}{0.0197} = 0.4975$$

The probability that a person has COVID if that person tested positive for COVID is only 49.7%! This means that there is a more than 50% chance that you don't have COVID even if you test positive for COVID.

5. Discrimination in the workplace is illegal, and companies that discriminate are often sued. The female instructors at a large university recently lodged a complaint about the most recent round of promotions from assistant professor to associate professor. An analysis of the relationship between gender and promotion produced the following joint probabilities.

	Promoted	Not Promoted
Female	.03	.12
Male	.17	.68

a. What is the rate of promotion among female assistant professors?

$$P(\text{promoted} \mid \text{female}) = \frac{P(\text{promoted and female})}{P(\text{female})} = \frac{.03}{.03+.12} = .20$$

b. What is the rate of promotion among male assistant professors?

$$P(\text{promoted} \mid \text{male}) = \frac{P(\text{promoted and male})}{P(\text{male})} = \frac{.17}{.17+.68} = .20$$

c. What is the probability of being a male given that one is promoted?

$$\frac{P(\text{promoted / male})\text{ x }P(\text{male})}{P(\text{promoted})} = \frac{.20(.85)}{.20} = .85$$

$$P(\text{male} \mid \text{promoted}) = \text{ or } \frac{P(\text{male} \cap \text{promoted}}{P(\text{promoted})} = \frac{.17}{.20} = .85$$

Using this formula P(A1/B1= (P(A1)P(B1/A1))/(P(A1)P(B1/A1)+P(A2)P(B1/A2))

Or

P(Male/Promoted) =

(P(Male)P(Promoted/Male)/(P(Male)P(Promoted/Male)+P(Female)P(Promoted/Female))

= (.85)(.20)/(.85(.20)+(.15)(.20) = .85