Approximating p-Values from the Student t Table

In Chapter 11 we showed how to calculate the p-value of a test when the sampling distribution is normal. The normal table is designed to calculate probabilities. However, in order to compute critical values z_A we use the table backward. The exact opposite is true for the student t distribution. The t-student table provides critical values t_A for a variety of degrees of freedom. However, in almost all cases we cannot compute probabilities from this table. In this document we show how to approximate p-values from the t-student.

Example 1

A statistics practitioner would like to determine whether there is sufficient evidence to infer that a population mean is greater than 25 given that in a sample of 9 observations $\bar{x} = 30$ and s = 6. Find the p-value of the test.

Solution

The null and alternative hypotheses are

$$H_0 : \mu = 25$$

$$H_1 : \mu > 25$$

The value of the test statistic is

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{30 - 25}{6 / \sqrt{9}} = 2.5$$

The p-value is

p-value = P(t > 2.5)

To calculate this probability we turn to Table 4. The first step is to determine the number of degrees of freedom. It is

$$\nu = n - 1 = 9 - 1 = 8$$

Next we find in Table 4 the row representing 8 degrees of freedom. The critical values of t are

$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{0.020}$	$t_{.010}$	$t_{.005}$
1.397	1.860	2.306	2.449	2.896	3.355

If a variable is Student t distributed with 8 degrees of freedom we can see that

$P(t > 2.449) = .020$

$P(t > 2.896) = .010$

We chose these values because the t-statistic is equal to 2.5, which lies between 2.449 and 2.896. Thus, we know that the p-value we seek lies between .010 and .020. To approximate this probability, we use *linear interpolation*. Table 1 depicts the location of the approximate p-value (denoted as $\hat{p}$ -value) between .010 and .020

Table 1 Approximating P(t > 2.500)

.020	$\hat{p}$ -value	.010
2.449	2.500	2.896

The difference between 2.896 and 2.449 is 2.896 – 2.449 = .447. The difference between 2.896 and 2.500 2.896 – 2.500 = .396. The ratio of these two numbers is .396/.447 = .8859. That is, the difference between 2.896 and 2.500 is .8859 of the difference between 2.896 and 2.449. We now apply this ratio to the probabilities. The difference between the $\hat{p}$ -value and .010 as a proportion of the difference between .020 and .010 is equal to the difference between 2.896 and 2.500 as a proportion of the difference between 2.896 and 2.449. That is,

$$\frac{\hat{p} - value - 0.010}{0.020 - 0.010} = \frac{2.896 - 2.50}{2.896 - 2.449} = \frac{0.396}{0.447} = 0.8859$$

which becomes

$$\frac{\hat{p} - value - 0.010}{0.010} = 0.8859$$

Solving this equation yields

$$\hat{p} - value = 0.8859\ (0.010) + 0.010 = 0.0188$$

We estimate the p-value to be .0188. Note that the exact value determined from Excel is .0185. (close enough)

Note that had we conducted a two-tail test where the hypotheses are

$H_0 : \mu = 25$

$H_1 : \mu \neq 25$

the p-value would be estimated to be 2(.0188) = 0.0376

If the value of the test statistic is greater than $t_{.005}$ we cannot use linear interpolation. However, it is not necessary because the one-tail (right-tail) p-value will be less than .005, which is small enough to conclude that there is enough evidence to support the alternative hypothesis. If the test is two-tail the p-value will be less than 2(.005) = .01.

If the test statistic is less than $t_{.100}$ the p-value will be at least .10, which we would judge to be too large to conclude that the alternative hypothesis is true.