Determining Equilibrium Income- The Simple Keynesian Model

We begin with the original Aggregate Expenditures or AD with prices fixed.

AE =AD = C + I + G + NX   In equilibrium: Y = AE

Thus,   Y = C + I + G + NX

To simplify our model, we will make a number of assumptions.

1. There is no foreign sector or X = 0 and M = 0.
2. Investment (I) is constant over some period.
3. Let T and G = 0
4. Prices are constant

SOLVING FOR EQUILIBRIUM INCOME

Now that all the components of Aggregate Expenditures are defined, the model can be solved for equilibrium income. Of course, the model will become more complicated as more variables are added.

To solve for equilibrium, we need the separate demand equations:

a) Consumption   C = (MPC or b)YD   YD = disposable income = Y since T = 0

b) Investment   I = I (constant)

Using the demand equations we can now solve for the equilibrium level of income:

SOLVING FOR EQUILIBRIUM

Y = C + I   or   Y = b(Y) + I   therefore   Y - bY = I   or Y(1 - b) = I

Solving for Y yields $Y = \dfrac{I}{1-b}$

Assume for simplicity that I = 100 and b = MPC = 0.8

$$Y = \frac{I}{1-b} = \frac{100}{1-0.8} = \frac{100}{0.2} = 500$$

Solve for Savings: $S = (1-b)Y = (1-0.8)(500) = 0.2(500) = 100$

THE MULTIPLIER

We can now examine what happens to equilibrium income if there is a change in one of the factors affecting Aggregate Expenditures.

It will be shown that an increase in investment will have magnified effect on income. This is known as the **MULTIPLIER EFFECT**.

Example: From before

$$Y = \frac{I}{1-b} = \frac{100}{1-0.8} = \frac{100}{0.2} = 500$$

If I increases by 100, how much does Y change?

$$Y = \frac{I}{1-b} = \frac{200}{1-0.8} = \frac{200}{0.2} = 1000$$

Notice that an increase in I by 100 causes Y to rise by 500 or five times the original change in the value of investment.

MULTIPLIER EFFECT

We know if ⇑ I --- Increase in autonomous investment -- ⇑ Y
and as Y⇑ --- Induced increase in consumption ⇑C -- ⇑ Y

MULTIPLIER EFFECT: An autonomous increase in consumption spending (a) or an increase in investment spending (I) will cause Y to rise. This increase in Y will cause
an induced increase in consumption which causes income to rise. This increase in income will cause a induced increase in consumption, and so on and so on.

⇑ I or --- ⇑Y --- ⇑C --- ⇑ Y --- ⇑C --- ⇑Y

Result: Equilibrium income increases more than the original increase in a, I or G.

Multiplier Effect, Step-by-Step

Look at the model above. Now suppose investment increases by 100 or $\Delta I = 100$. Initial change in I causes Y to change by 100.

Round	**ΔY**	**ΔC**	**ΔS**
1	100	80	20
2	80	.8(80) = 64	16
3	64	.8(64) = 51.2	12.8
4	"	"	"
5	"	"	"
	500	400	100

Adding up all the rounds yields:

Total $\Delta Y = \Delta I + \Delta I\,(0.8) + \Delta I\,(0.8)^2 + \Delta I\,(0.8)^3 + \ldots + \Delta I\,(0.8)^\infty$
Total $\Delta Y = \Delta I + \Delta I\,(b) + \Delta I\,(b)^2\,\Delta I\,(b)^3 + \ldots + \Delta I\,(b)^\infty$
Total $\Delta Y = \Delta I\,(1 + b + b^2 + b^3 + \ldots + b^\infty)$

We want to solve for the value of $(1 + b + b^2 + b^3 + \ldots + b^\infty)$.

Define $X = (1 + b + b^2 + b^3 + \ldots + b^\infty)$. Multiply both sides by $(1 - b)$

$X\,(1 - b) = (1 + b + b^2 + b^3 + \ldots + b^\infty)\,(1 - b)$.
$X\,(1 - b) = (1 + b + b^2 + b^3 + \ldots + b^\infty - b - b^2 - b^3 - \ldots - b^\infty) = X\,(1 - b) = 1$

Solving for X: $X = \dfrac{1}{1-b}$ We can then define the multiplier as

$$\alpha_I = \frac{1}{1-b}$$

$$\Delta Y = \Delta I\left(\frac{1}{1-b}\right) \quad or\ \Delta Y = \Delta I\,\alpha_I$$

Here is an example where the b or MPC = 0.75 and the change in I or $\Delta I = 5$.

	(1) Change in Income	(2) Change in Consumption (MPC = .75)	(3) Change in Saving (MPS = .25)
Increase in investment of **\$5.00**	\$5.00	\$ 3.75	\$1.25
Second round	3.75	2.81	.94
Third round	2.81	2.11	.70
Fourth round	2.11	1.58	.53
Fifth round	1.58	1.19	.39
All other rounds	4.75	3.56	1.19
Total	**\$20.00**	\$15.00	\$5.00